Continue

Chain rule derivative examples

The chain rule is used to calculate the derivative of a composite function. The chain rule formula states that dy/dx = dy/du × du/dx. In words, differentiate the outer function while keeping the inner function the same then multiply this by the derivative of the inner function. The chain rule is used when a function is within another function. The chain rule is defined as, where u is a function of x (u = g(x)) and y is a function of u (y = f(u)). Alternatively, the chain rule can be written in function notation as F'(x) = f'(g(x)).g'(x), where F(x) = f(g(x)). g(x) is the inner function and f(x) is the outer function. In words, the chain rule requires finding the derivative of the outer function while keeping the inner function the same and then multiplying this by the derivative of the inner function. To use the chain rule, the following rules are required: The function must be a composite function of two or more functionsSuch functions must be differentiable themselves To do the chain rule: Differentiate the outer function, keeping the inner function the same. Multiply this by the derivative of the inner function. In this example we will use the chain rule step-by-step. Below this, we will use the chain rule formula method. We define 4x – 3 as the inner function and the ( )5 as the outer function. Step 1. Differentiate the outer function, keeping the inner function the same The outer function is the ( )5. We differentiate this like we would x5. x5 would differentiate to 5x4 and so we write ( )5 differentiated as 5( )4. We keep the inner function of 4x-3 the same, so we write 5(4x – 3)4. However because we have 4x – 3 inside the brackets and not just x, we must also include step 2. Step 2. Multiply this by the derivative of the inner function From step 1. we already have 5(4x – 3)4 and now we must multiply this by the derivative of the inner function. The inner function is the function inside the brackets. It is 4x – 3. We differentiate 4x – 3 to get 4. So we must multiply the result of 5(4x – 3)4 by 4. We arrive at the answer of f'(x) = 20(4x – 3)4. We will now use the chain rule formula to differentiate this function. The answer will be the same. The chain rule formula states that F'(x) = f'(g(x).g'(x), where g(x) is the inner function and f(x) is the outer function. Therefore for F(x) = (4x – 3)5, g(x) = 4x – 3 and f(x) = x5. f'(x) is therefore 5x4. f'(g(x)) means to substitute g(x) in place of x in f'(x). Therefore f'(g(x)) = 5(4x – 3)4. g'(x) = 4. Therefore F'(x) = f'(g(x).g'(x) becomes F'(x) = 5(4x – 3)4 × 4. This simplifies to F'(x) = 20(4x – 3)4. The chain rule is used to differentiate trigonometric functions containing another function. Differentiate the trigonometric function, keeping the inner function the same and then multiply this by the derivative of the inner function. The following rules for differentiating the trigonometric functions of sin(x), cos(x) and tan(x) may be useful here: cos(2x) can be written as an inner function of 2x and an outer function of cos(). Step 1. Differentiate the outer function, keeping the inner function the same cos(x) differentiates to -sin(x) and so, keeping the inner function as 2x rather than x, we get -sin(2x). Step 2. Multiply this by the derivative of the inner function The inner function is 2x and its derivative is 2. We multiply -sin(2x) by 2 to get f'(x) = -2sin(2x). The chain rule can be applied to trigonometric functions raised to a power. Write the trigonometric function as the inner function in brackets and the power as the outer function. Bring down the power and subtract one from the power, keeping the trigonometric function inside the same. Multiply this by the derivative of the trigonometric function. Step 1. First write sin4(2x) as [sin(2x)]4. Step 2. Bring down the power and subtract one from the power, keeping the trigonometric function the same. We bring down the 4 in front of the brackets, the power becomes 3 and we keep sin(2x) inside the brackets, which gives us 4[sin(2x)]3. Step 3. Multiply by the derivative of sin(2x). We use the chain rule here, so sin(x) is the outer function which differentiates to cos(x). We then multiply this by the derivative of 2x, which is 2. sin(2x) differentiated is 2cos(2x). So we multiply 4[sin(2x)]3 by 2cos(2x). f'(x) = 8sin3(2x)cos(2x) The chain rule states that for y = ln(u), dy/dx = 1/u × du/dx. In words, differentiate the inner function and then divide this by the inner function. For example if y = ln(x2 + 3x), dy/dx = (2x + 3)/(x2 + 3x). The derivative of y = ln(u) is 1/u × du/dx. u is the function inside the ln function. For y = ln(x2 + 3x), u = x2 + 3x. Therefore du/dx = 2x + 3. The derivative dy/dx =. We can write this as . The chain rule allows us to differentiate log functions. The derivative of y = ln(f(x)) can be written as: or . To differentiate log of any base, if y = logaf(x) then . For example, differentiate y = log5(2x + 1): Here f(x) = 2x + 1 and so, f'(x) = 2. Substituting these into the formula, we get: The derivative of y = ex is dy/dx = ex and so using the chain rule, the derivative of y = ef(x) is dy/dx = f'(x).ef(x). Simply differentiate the power of e and multiply this by the original function. For example, differentiate e5x+3. In this example, f(x) = 5x + 3 and so, f'(x) = 5. To find the derivative of y = e5x+3 , we simply multiply it by 5. We get dy/dx = 5 e5x+3. The chain rule can be applied to the composition of three functions. If y(x) = h(g(f(x))), then y'(x) = f'(x) . g'(f(x)) . h'(g(f(x))). However, it is easier to apply the chain rule twice to differentiate three functions. For example, differentiate f(x) = sin2(5x). Here we have the composition of 3 functions. The innermost function is 5x.Acting on the 5x is the sin functionFinally, the sin function is raised to the power of 2 We write sin2(5x) as [sin(5x)]2. We differentiate using the chain rule, so that we differentiate the outer function, keeping its inner function the same and then multiply by the derivative of the inner function. Differentiating the outer squared function, keeping the inner function the same we have 2[sin(5x)]. The power 2 came down, we subtracted 1 from the power and we kept the sin(5x) on the inside the same. We now need to multiply this by the derivative of the inner function, sin(5x). This requires the chain rule to be used for a second time. We differentiate the outer function of sin to get cos and we keep the inner function of 5x the same. We get cos(5x). We need to multiply this by the derivative of the inner function, 5x. We find the derivative of sin(5x) is 5cos(5x). We now multiply the previous result of 2[sin(5x)] by 5cos(5x) to get the final derivative of 10sin(5x)cos(5x). Here are some examples of using the chain rule to differentiate a variety of functions: FunctionCalculationDerivativef(x) = (5x + 1)33 × (5x + 1)2 × 5 f'(x) = 15(5x + 1)2 f(x) = sin(5x) cos(5x) × 5 f'(x) = 5cos(5x)f(x) = cos(10x) -sin(10x) × 10f'(x) = -10sin(10x) f(x) = -3tan(2x) -3sec2(2x) × 2f'(x) = -6 sec2(2x) f(x) = 10e4x 10e4x × 4 f'(x) = 40e4x f(x) = ln(x3) 1/(x3) × 3x2 = (3x2)/(x3) f'(x) = 3/x The chain rule is used to differentiate any composite function of the form y = f(g(x)). That is a function that has an inner function with an outer function applied to it. For example y = (3x + 2)5 is made of the functions g(x) = 3x + 2 and f(x) = x5. Here are some example of when and when not to use the chain rule: ExampleShould I use the chain rule?Why?y = (3x+2)5 YesThe inner function is 3x+2 and the outer function is x5. y = x5 NoThere is only one function x5. It can be differentiated directly.y = (x+3) sin(x) NoIt is the product of two functions. Use the product rule.y = sin(2x+1)YesThe inner function is 2x+1 and the outer function is sin(x) y = (x+5)/(3x+5)NoIt is a fraction with a function for a numerator and denominator. Use the quotient rule.y = ln(x4-x) YesThe inner function is x4-x. The outer function is ln(x). y = e2x YesThe inner function is 2x and the outer function is ex. To prove the chain rule, consider dy/dx as a limit of Δy/Δx as x tends to zero. This can be written as limits of Δy/Δu × Δu/Δx. Evaluating these limits as x and u tend to zero, we derive the chain rule as dy/dx = dy/du × dy/dx. To prove the chain rule: Write dy/dx as a limit of Δy/Δx as x tends to zero.Write the limit of Δy/Δx as Δy/Δu × Δu/Δx. Δu is just a number so this can be done.Separate the limit into 2 limits of Δy/Δu and Δu/Δx.As Δx tends to 0, so does Δu.Evaluate both limits to get dy/dx = dy/du × du/dx. The following is a short summary of the key rules for calculating derivatives. Keep in mind that knowing how to calculate derivatives is useless unless you know what they are and how to use them. There are two types of formulas for calculating derivatives, which we may classify as Power Rule: d/du (un) = nun-1 d/du (ln u) = 1/u d/du (eu) = eu d/du (sin u) = cos u d/du (cos u) = - sin u d/du (tan u) = sec 2 u Note that these formulas may be used only to calculate derivatives of functions in precisely the form indicated. The name of the independent variable is irrelevant--it is the form that is important. For example, the power rule may be used to calculate that d/dx (x5) = 5x4 or that d/du (u5) = 5u4, but gives us no help at all with d/du (x5). All other formulas, the structural type formulas, reduce derivatives of more complicated functions until we merely need to apply the above formulas a number of times. These formulas may be divided into two groups; one group is used almost without thought, while the other group needs to be carefully memorized. In the following, assume that x, y, u, v and t are variables, while c and k are constants. The first group of formulas, which is used almost without thought, may be expressed as: The derivative of a constant times a function equals the contant times the derivative of the function. The derivative of a sum equals the sum of the derivatives. The derivative of a difference equals the difference of the derivatives. Symbolically, we write these rules as: d/dx (cu) = c du/dx d/dx (u+v) = du/dx + dv/du d/dx (u-v) = du/dx - dv/du. When we apply these rules, we say that we are differentiating "term by term." Using these rules along with the power rule, it is very easy to differentiate any polynomial. Some special cases such as the following come up so often that we tend to take them for granted: d/du (c) = 0 d/du (cu) = c d/du (au+b) = a The last three rules are somewhat more difficult. They are called the product rule, the quotient rule and the chain rule. Of these, the product and quotient rules can be used routinely, since it is easy to recognize when you have a product or quotient, but it is more difficult and takes more practice to use the chain rule correctly. The product rule may be thought of as the derivative of a product equals the first factor times the derivative of the second plus the second factor times the derivative of the first. The quotient rule may be thought of as the derivative of a quotient equals the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator. Symbolically, we express these rules as follows: Product Rule: d/dx (uv) = u dv/dx + v du/dx Quotient Rule: d/dx (u/v) = (v du/dx - u dv/dx)/v2 However, we should remember the rules in words rather than trying to memorize jumbles of symbols. The chain rule, as mentioned before, is a little trickier to use. Fortunately, its formula is easier to remember than some of the others. Chain Rule: dy/dx = (dy/du)· (du/dx) As usual, remember the meaning, especially since the symbols x,y and u are not necessarily the ones that will be used in the function you will be differentiating. The chain rule is used for calculating the derivatives of composite functions. The easiest way to recognize that you are dealing with a composite function is by the process of elimination: if none of the other rules apply, then you have a composite function. This makes it very important to be totally familiar with the other rules for differentiation. The difficult part of using the chain rule is recognizing that you need it and then doing the algebra necessary to decompose the function. We have looked at two ways of doing that in class. Study them and, if you have difficulty, see your instructor. Once you have decomposed the function, so that you have written down something in the form y = f(u) u = g(x), it is easy to actually carry out the rest of the chain rule. For example, given the function y=√(x2 + 5) you would write something like y = u1/2 u = x2 + 5 and then, since d/du(u1/2) = (1/2)u-1/2 and d/dx(x2 + 5) = 2x, we get dy/dx = (dy/du) · (du/dx) = (1/2)u-1/2 · 2x = x/√(x2 + 5). The following is a general approach to calculating derivatives which, if followed, will enable you to routinely calculate every derivative we run across in the class. It depends on your being able to recognize individual terms, constant factors, products and quotients and also being able to rewrite functions in a more useful style, such as being able to recognize that something like x/50 can be rewritten as (1/50) ·x or that √x can be rewritten as x1/2. Differentiate term by term. Deal with each term separately and, for each term, recognize any constant factor. For each term, recognize whether it is one of the elementary functions - power, trigonometric, exponential or natural logarithm of the independent variable. If it is, you can easily apply the appropriate formula and you will be done. If it's not, continue. Decide whether the term is a product or a quotient. If it is, use the appropriate formula. Note that the appropriate formula will have you calculating two other derivatives and you will have to go back to the beginning to deal with those. If the term isn't a product or a quotient, you will need to use the Chain Rule. Try to use it correctly. The process of differentiation is, for most of the functions encountered in a lower division calculus class, a very mechanical process. With sufficient practice, reliably calculating derivatives should and will take no more mental effort than doing ordinary arithmetic. It's important to reach that level of expertise so that you can expend your mental energy on more significant matters. If you have difficulty decomposing a composite function into components, consider the following two methods which, together, work on almost all composite functions ordinarily encountered in elementary calculus. Shown below is how each method works on the same function, y = sin(x2). In this method, think of a function as a black box which takes an input, the value of the independent variable, and produces an output, the value of the function corresponding to that value of the independent variable. To use this method, try to break up the black box into two separate black boxes, the second of which takes the output of the first as its input. This can generally be done by carefully considering how one would calculate the function for a particular value of the independent variable. Consider how one would evaluate y = sin(x2) for a particular value of x, say for x = 3. Most people would say that the value would be sin(9), and might then take out a calculator (a prototypical black box) to find a decimal approximation to sin(9). If one thinks about what was actually done, one took the input, 3, and came up with an output value of 9 by squaring 3. One then takes 9 as input and comes up with the output of sin(9). One can see that, in general, one would calculate y by taking x, squaring it to get x2, and then taking x2 and calculating its sine. Symbolically, x --> x2 --> sin(x2) = y. If one represents the initial output, x2, by the symbol u, one can write x --> x2 = u --> sin(u) = y. By reading right to left, one comes up with the decomposition y = sin(u), u = x2. The Walt Disney Method is based upon the idea of making one's wish come true. Often one will look at a composite function and, at least subliminally, wish it were something similar but somewhat simpler. With this method, one expediently but mathematically legally makes that wish come true - at least partially. Consider again the function y = sin(x2). One might look at that and wish that it was the simpler function y = sin(x). One makes that wish almost come true by writing it as y = sin(u), which one can legally do by letting u = x2. Looking at that again, one has come up with the decomposition y = sin(u), u = x2, exactly the same one generated through the use of the Black Box Method. The Walt Disney Method makes use of the general principles that one can always assign a symbol to represent anything on wants, provided the symbol isn't already in use, and one can always replace any expression with anything else equal to it, the substitution principle. In general, one implements the Walt Disney Method by looking at the form of a function y = f(x) and wistfully noticing that it would look like a much simpler function, say g(x), which would be much easier to differentiate were some part of it just x. It's also important that the independent variable x not appear outside of that part. One calls that part u and has the decomposition y = g(u), u = h(x), where h(x) is the part that one wished was just x. Last modified by Alan H. Stein. The Chain Rule, as learned in Section 2.5, states that ddx(f(g(x)))=f′(g(x))g′(x). If t=g(x), we can express the Chain Rule as In this section we extend the Chain Rule to functions of more than one variable. Let z=f(x,y), x=g(t) and y=h(t), where f, g and h are differentiable functions. Then z=f(x,y)=f(g(t),h(t)) is a function of t, and dzdt=dfdt=fx(x,y)dxdt+fy(x,y)dydt=∂f∂xdxdt+∂f∂ydydt. Proof By definition, dfdt(x,y)=limh→0f(x(t+h),y(t+h))-f(x,y)h. Let Δf =f(x(t+h),y(t+h))-f(x,y), dx =x(t+h)-x(t),and dy =y(t+h)-y(t). Because f is differentiable, Definition 13.4.2 gives us functions E1 and E2 so that E1dx+E2dy=Δf-fx(x,y)dx-fy(x,y)dy, lim(dx,dy)→0E1=0,and lim(dx,dy)→0E2=0. This means that dfdt(x,y) =limh→0fx(x,y)dx+fy(x,y)dy+E1dx+E2dyh =fx(x,y)limh→0dxh+fy(x,y)limh→0dyh +limh→0E1limh→0dxh+limh→0E2limh→0dyh =fx(x,y)x′(t)+fy(x,y)y′(t)+0x′(t)+0y′(t).∎ It is good to understand what the situation of z=f(x,y), x=g(t) and y=h(t) describes. We know that z=f(x,y) describes a surface; we also recognize that x=g(t) and y=h(t) are parametric equations for a curve in the x-y plane. Combining these together, we are describing a curve that lies on the surface described by f. The parametric equations for this curve are x=g(t), y=h(t) and z=f(g(t),h(t)). ††margin: (fullscreen) Figure 13.5.1: Understanding the application of the Multivariable Chain Rule. Λ Consider Figure 13.5.1 in which a surface is drawn, along with a dashed curve in the x-y plane. Restricting f to just the points on this circle gives the curve shown on the surface. The derivative dfdt gives the instantaneous rate of change of f with respect to t. If we consider an object traveling along this path, dfdt gives the rate at which the object rises/falls. We now practice applying the Multivariable Chain Rule. Let z=x2y+x, where x=sint and y=e5t. Find dzdt using the Chain Rule. SolutionFollowing Theorem 13.5.1, we find fx(x,y)=2xy+1,fy(x,y)=x2,dxdt=cost,dydt=5e5t. Applying the theorem, we have dzdt=(2xy+1)cost+5x2e5t. This may look odd, as it seems that dzdt is a function of x, y and t. Since x and y are functions of t, dzdt is really just a function of t, and we can replace x with sint and y with e5t: dzdt=(2xy+1)cost+5x2e5t=(2sin(t)e5t+1)cost+5e5tsin2t. The previous example can make us wonder: if we substituted for x and y at the end to show that dzdt is really just a function of t, why not substitute before differentiating, showing clearly that z is a function of t? That is, z=x2y+x=(sint)2e5t+sint. Applying the Chain and Product Rules, we have dzdt=2sintcoste5t+5sin2te5t+cost, which matches the result from the example. This may now make one wonder “What’s the point? If we could already find the derivative, why learn another way of finding it?” In some cases, applying this rule makes deriving simpler, but this is hardly the power of the Chain Rule. Rather, in the case where z=f(x,y), x=g(t) and y=h(t), the Chain Rule is extremely powerful when we do not know what f, g and/or h are. It may be hard to believe, but often in “the real world” we know rate-of-change information (i.e., information about derivatives) without explicitly knowing the underlying functions. The Chain Rule allows us to combine several rates of change to find another rate of change. The Chain Rule also has theoretic use, giving us insight into the behavior of certain constructions (as we’ll see in the next section). We demonstrate this in the next example. An object travels along a path on a surface. The exact path and surface are not known, but at time t=t0 it is known that : ∂z∂x=5,∂z∂y=-2,dxdt=3 and dydt=7. Find dzdt at time t0. SolutionThe Multivariable Chain Rule states that dzdt =∂z∂xdxdt+∂z∂ydydt =5(3)+(-2)(7) =1. By knowing certain rates-of-change information about the surface and about the path of the particle in the x-y plane, we can determine how quickly the object is rising/falling. We next apply the Chain Rule to solve a max/min problem. Consider the surface z=x2+y2-xy, a paraboloid, on which a particle moves with x and y coordinates given by x=cost and y=sint. Find dzdt when t=0, and find where the particle reaches its maximum/minimum z-values. SolutionIt is straightforward to compute fx(x,y)=2x-y,fy(x,y)=2y-x,dxdt=-sint,dydt=cost. Combining these according to the Chain Rule gives: dzdt=-(2x-y)sint+(2y-x)cost. When t=0, x=1 and y=0. Thus dzdt=-(2)(0)+(-1)(1)=-1. When t=0, the particle is moving down, as shown in Figure 13.5.2. ††margin: (fullscreen) Figure 13.5.2: Plotting the path of a particle on a surface in Example 13.5.3. Λ To find where z-value is maximized/minimized on the particle’s path, we set dzdt=0 and solve for t: dzdt=0 =-(2x-y)sint+(2y-x)cost 0 =-(2cost-sint)sint+(2sint-cost)cost 0 =sin2t-cos2t cos2t =sin2t t =nπ4 (for odd n) We can use the First Derivative Test to find that on [0,2π], z has reaches its absolute minimum at t=π/4 and 5π/4; it reaches its absolute maximum at t=3π/4 and 7π/4, as shown in Figure 13.5.2. We can extend the Chain Rule to include the situation where z is a function of more than one variable, and each of these variables is also a function of more than one variable. The basic case of this is where z=f(x,y), and x and y are functions of two variables, say s and t. (a) Let z=f(x,y), x=g(s,t) and y=h(s,t), where f, g and h are differentiable functions. Then z is a function of s and t, and • ∂z∂s=∂f∂x∂x∂s+∂f∂y∂y∂s , and • ∂z∂t=∂f∂x∂x∂t+∂f∂y∂y∂t. (b) Let z=f(x1,x2,…,xm) be a differentiable function of m variables, where each of the xi is a differentiable function of the variables t1,t2,…,tn. Then z is a function of the ti, and ∂z∂ti=∂f∂x1∂x1∂ti+∂f∂x2∂x2∂ti+⋯+∂f∂xm∂xm∂ti. The proof of Part II follows quickly from Part I, because ∂∂ti means that we hold the other variables constant and we are back to the one variable case already proved. A helpful way to remember the derivatives is to examine the following chart zx1x2⋯xmt1t2⋯tn Each possible path from f to the variable ti contributes a term to the sum, and each line segment in a path contributes a factor to that term. Watch the video: Generalized Chain Rule — Part 1 from Let z=x2y+x, x=s2+3t and y=2s-t. Find ∂z∂s and ∂z∂t, and evaluate each when s=1 and t=2. SolutionFollowing Theorem 13.5.2, we compute the following partial derivatives: ∂f∂x=2xy+1 ∂f∂y=x2, ∂x∂s=2s ∂x∂t=3 ∂y∂s=2 ∂y∂t=-1. Thus ∂z∂s=(2xy+1)(2s)+(x2)(2)=4xys+2s+2x2,and ∂z∂t=(2xy+1)(3)+(x2)(-1)=6xy-x2+3. When s=1 and t=2, x=7 and y=0, so ∂z∂s=100 and ∂z∂t=-46. Let w=xy+z2, where x=t2es, y=tcoss, and z=ssint. Find ∂w∂t when s=0 and t=π. SolutionFollowing Theorem 13.5.2, we compute the following partial derivatives: ∂f∂x=y ∂f∂y=x ∂f∂z=2z, ∂x∂t=2tes ∂y∂t=coss ∂z∂t=scost. Thus ∂w∂t=y(2tes)+x(coss)+2z(scost). When s=0 and t=π, we have x=π2, y=π and z=0. Thus We studied finding dydx when y is given as an implicit function of x in detail in Section 2.6. We find here that the Multivariable Chain Rule gives a simpler method of finding dydx. For instance, consider the implicit function x2y-xy3=3. We learned to use the following steps to find dydx: ddx(x2y-xy3) =ddx(3) 2xy+x2dydx-y3-3xy2dydx =0 dydx=-2xy-y3x2-3xy2. (13.5.1) Instead of using this method, consider z=x2y-xy3. The implicit function above describes the level curve z=3. Considering x and y as functions of x, the Multivariable Chain Rule states that dzdx=∂z∂xdxdx+∂z∂ydydx. (13.5.2) Since z is constant (in our example, z=3), dzdx=0. We also know dxdx=1. Equation (13.5.2) becomes 0 =∂z∂x(1)+∂z∂ydydx dydx =-∂z∂x/∂z∂y =-fxfy. Note how our solution for dydx in Equation (13.5.1) is just the negative of the partial derivative of z with respect to x, divided by the partial derivative of z with respect to y. We state the above as a theorem for two and three variables. If f is a differentiable function of x and y, where f(x,y)=c defines y as an implicit function of x for some constant c, then dydx=-fx(x,y)fy(x,y). If f is a differentiable function of x, y, and z, where f(x,y,z)=c defines z as an implicit function of x and y for some constant c, then ∂z∂x=-fx(x,y,z)fz(x,y,z) and ∂z∂y=-fy(x,y,z)fz(x,y,z). Given the implicitly defined function sin(x2y2)+y3=x+y, find y′. Note: this is the same problem as given in Example 2.6.4 of Section 2.6. SolutionLet f(x,y)=sin(x2y2)+y3-x-y; the implicitly defined function above is equivalent to f(x,y)=0. We find dydx by applying Theorem 13.5.3. We find fx(x,y)=2xy2cos(x2y2)-1 and fy(x,y)=2x2ycos(x2y2)+3y2-1, so dydx=-2xy2cos(x2y2)-12x2ycos(x2y2)+3y2-1, which matches our solution from Example 2.6.4. 1. Let a level curve of z=f(x,y) be described by x=g(t), y=h(t). Explain why dzdt=0. 2. Fill in the blank: The single variable Chain Rule states ddx(f(g(x)))=f′(g(x))· . 3. Fill in the blank: The Multivariable Chain Rule states that if f is a function of x and y which are each a function of t, then dfdt=∂f∂x· + ·dydt. 4. If z=f(x,y), where x=g(t) and y=h(t), we can substitute and write z as an explicit function of t. T/F: Using the Multivariable Chain Rule to find dzdt is sometimes easier than first substituting and then taking the derivative. 5. T/F: The Multivariable Chain Rule is only useful when all the related functions are known explicitly. 6. The Multivariable Chain Rule allows us to compute implicit derivatives easily by just computing two derivatives. In Exercises 7–12., functions z=f(x,y), x=g(t) and y=h(t) are given. (a) Use the Multivariable Chain Rule to compute dzdt. (b) Evaluate dzdt at the indicated t-value. 7. z=3x+4y, x=t2, y=2t; t=1 8. z=x2-y2, x=t, y=t2-1; t=1 9. z=5x+2y, x=2cost+1, y=sint-3; t=π/4 10. z=xy2+1, x=cost, y=sint; t=π/2 11. z=x2+2y2, x=sint, y=3sint; t=π/4 12. z=cosxsiny, x=πt, y=2πt+π/2; t=3 In Exercises 13–18., functions z=f(x,y), x=g(t) and y=h(t) are given. Find the values of t where dzdt=0. Note: these are the same surfaces/curves as found in Exercises 7.–12.. 13. 14. 15. z=5x+2y, x=2cost+1, y=sint-3 16. z=xy2+1, x=cost, y=sint 17. z=x2+2y2, x=sint, y=3sint 18. z=cosxsiny, x=πt, y=2πt+π/2 In Exercises 19–22., functions z=f(x,y), x=g(s,t) and y=h(s,t) are given. (a) Use the Multivariable Chain Rule to compute ∂z∂s and ∂z∂t. (b) Evaluate ∂z∂s and ∂z∂t at the indicated s and t values. 19. z=x2y, x=s-t, y=2s+4t; s=1, t=0 20. z=cos(πx+π2y), x=st2, y=s2t; s=1, t=1 21. z=x2+y2, x=scost, y=ssint; s=2, t=π/4 22. z=e-(x2+y2), x=t, y=st2; s=1, t=1 23. 24. 25. 26. In Exercises 27–30., find dzdt, or ∂z∂s and ∂z∂t, using the supplied information. 27. ∂z∂x=2, ∂z∂y=1, dxdt=4, dydt=-5 28. ∂z∂x=1, ∂z∂y=-3, dxdt=6, dydt=2 29. ∂x∂s=5, ∂x∂t=7, ∂y∂s=-2, ∂y∂t=6 30. ∂x∂s=-2, ∂x∂t=3, ∂y∂s=2, ∂y∂t=-1 31. Suppose z=f(x,y) is differentiable. Express (x,y) in polar coordinates as x=rcosθ, y=rsinθ. Calculate ∂z∂θ. 32. Suppose w=g(x,y,z) is differentiable. Express (x,y,z) in spherical coordinates as x=ρsinϕcosθ, y=ρsinϕsinθ, z=ρcosϕ. Calculate ∂w∂ϕ. 33. Suppose the radius of a circular cylinder is increasing at the constant rate of 1/2 cm/sec while its height is decreasing at the rate of 1/3 cm/sec. How is the volume of the cylinder changing when the radius 4 cm and the height is 10 cm? Generated on Sun Nov 21 19:48:25 2021 by LaTeXML